

$$2\pi v' = 2\pi v - \left(\frac{2\pi}{\lambda}\right)v$$

$$\sqrt{1 - v^2/c^2}$$

$$v' = \frac{v - \frac{v}{\lambda}}{\sqrt{1 - v^2/c^2}}$$

$$\left[\begin{array}{l} \text{As } c = v\lambda \\ \frac{1}{\lambda} = \frac{v}{c} \end{array} \right]$$

$$v' = \frac{v - v \times \frac{v}{c}}{\sqrt{1 - v^2/c^2}}$$

$$v' = v \left(1 - \frac{v}{c}\right) \sqrt{\left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right)}$$

$$v' = v \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}$$

$$= v \frac{\sqrt{\frac{c-v}{c}}}{\sqrt{\frac{c+v}{c}}}$$

$$v' = v \sqrt{\frac{c-v}{c+v}}$$

$$\text{As } c = v\lambda = v'\lambda'$$

$$v' = \frac{v\lambda}{\lambda'}$$

$$\frac{v\lambda}{\lambda'} = v \sqrt{\frac{c-v}{c+v}}$$

$$\therefore \lambda' = \lambda \sqrt{\frac{c+v}{c-v}}$$